

Notes will be fill-in-the blanks (green words), where we will go through exercises before our students attempt the practice questions, and ask questions if they have difficulties.

## Sample Content: Ratio

### 1. Ratio Basics

Equivalent ratios are the ratios that are the same when we compare them.

- Examples of equivalent ratios:

$$2 : 3 = 4 : 6 = 6 : 9 = 20 : 30$$

- 2-Step equivalent ratios:

Some ratios need to be simplified first before getting the equivalent fraction.

$$40 : 30 = 16 : \underline{\quad}$$

$$40 : 30 = 4 : 3 = 16 : 12$$

#### Equivalent Ratios Practice:

1. Find the missing number.

(a) $6 : 15 = 4 : \underline{\quad}$ 6 : 15 2 : 5 4 : 10	(b) $9 : 15 = 12 : \underline{\quad}$ 9 : 15 3 : 5 12 : 20
(c) $15 : 35 = 9 : \underline{\quad}$ 15 : 35 3 : 7 9 : 21	(d) $21 : 35 = 12 : \underline{\quad}$ 21 : 35 3 : 5 12 : 20

#### Ratios and Simplifying Ratios:

Blue : Red : Yellow

10 : 5 : 20

2 : 1 : 4

#### Combining Ratios:

The ratio of A to B is 4 : 5. The ratio of B to C is 10 : 7. What is the ratio of A : B : C?

A : B                      B : C

4 : 5                      **10** : 7

8 : **10**

To combine ratios, make the ratio of B the same for both sides.

A : B : C

8 : 10 : 7

#### Ratios can also be written as fractions

If A is  $\frac{3}{4}$  of B  $\rightarrow A : B = 3 : 4$

If A is  $\frac{5}{2}$  of B  $\rightarrow A : B = 5 : 2$

If A is 1.5 of B, A is  $\frac{3}{2}$  of B  $\rightarrow A : B = 3 : 2$

## 2. Ratio Constant Part

Class Example: Constant Part

1. The ratio of Adam's savings to Ben's savings was 6 : 7.

Ben spent \$38 and the ratio of Adam's savings to Ben's savings became 8 : 3.

How much did Ben have at first?

Why is this called a constant part question? (Did Adam spend or gain any money? – no)

A	:	B	After Ben spent \$38	A	:	B
6	:	7	→	8	:	3
24	:	28	Make A the same	24	:	9

$$28u - 9u = 19u$$

$$19u = 38$$

$$1u = 38 \div 19$$

$$= 2$$

$$\text{Ben at first} = 28 \times 2$$

$$= \$56$$

## 3. Ratio Constant Total

Class Example: Constant Total

4. The ratio of Adam's savings to Ben's savings is 7 : 5.

After Adam gives \$22 to Ben, the ratio of Adam's savings to Ben's savings becomes 2 : 3.

How much did Ben have at first?

Why is this called a constant total question? (Did the total amount of money change? – no)

A	:	B	:	Total	After Adam gives \$22 to Ben	A	:	B	:	Total
7	:	5	:	12	→	2	:	3	:	5
35	:	25	:	60	Make Total Same	24	:	36	:	60

$$35u - 24u = 11u$$

$$11u = 22$$

$$1u = 22 \div 11$$

$$= 2$$

$$\text{Ben at first} = 25 \times 2$$

$$= \$50$$

#### 4. Ratio Everything Changed

Class Example: Everything Changed

10. Mrs Howard is 4 times as old as her son. In 14 years' time, her son will be  $\frac{1}{2}$  as old as her. How old is Mrs Howard now? (Everything Changed to Different Ratio – Algebra Method)

$$\begin{array}{ccc}
 \text{MrsH} & : & \text{Son} \\
 4 & : & 1
 \end{array}
 \quad
 \begin{array}{c}
 \text{After 14 years} \\
 \rightarrow
 \end{array}
 \quad
 \begin{array}{ccc}
 \text{MrsH} & : & \text{Son} \\
 2 & : & 1 \\
 4u+14 & : & 1u+14
 \end{array}$$

Using Algebra Manipulation,

$$(4u + 14) = (1u+14) \times 2$$

$$4u + 14 = 2u + 28$$

$$4u - 2u = 28 - 14 \rightarrow \text{Move 'smaller unit'}$$

$$2u = 14$$

$$1u = 14 \div 2$$

$$= 7$$

$$\begin{aligned}
 \text{Mrs Howard Now} &= 4u \\
 &= 4 \times 7 \\
 &= 28
 \end{aligned}$$

**Mrs Howard is 28 years old now.**

#### Practice Questions: Ratios

1. In a pet shop, the ratio of the number of hamsters to the number of rabbits is 11 : 2. After 18 of the hamsters were sold, the ratio of the number of hamsters to the number of rabbits became 4 : 1. How many hamsters and rabbits were there in the pet shop in the end?

**Constant Part** → Only 18 hamsters sold.

$$\begin{array}{ccc}
 \text{H} & : & \text{R} \\
 11 & : & 2
 \end{array}
 \quad
 \begin{array}{c}
 18 \text{ hamsters sold} \\
 \rightarrow \\
 \text{Make R the same}
 \end{array}
 \quad
 \begin{array}{ccc}
 \text{H} & : & \text{R} \\
 4 & : & 1 \\
 8 & : & 2
 \end{array}$$

$$11u - 8u = 3u$$

$$3u = 18$$

$$1u = 18 \div 3$$

$$= 6$$

$$\begin{aligned}
 \text{Total} &= 8u + 2u \\
 &= 10 \times 6 \\
 &= 60
 \end{aligned}$$

11. The ratio of number of 20-cent coins to the number of 50-cent coins in John's wallet was 2 : 7. After John spent 54 of the 50-cent coins, the ratio of the number of 20-cent coins to the number of 50-cent coins became 5 : 4. How much money did John have at the end.

**Constant Part** → Only spent 54 50-cent coins.

20-cent	:	50-cent	54 50-cent coins used	20-cent	:	50-cent
2	:	7	→	5	:	4
<b>10</b>	:	35	Make 20-cent the same	<b>10</b>	:	8

$$35u - 8u = 27u$$

$$27u = 54$$

$$1u = 54 \div 27$$

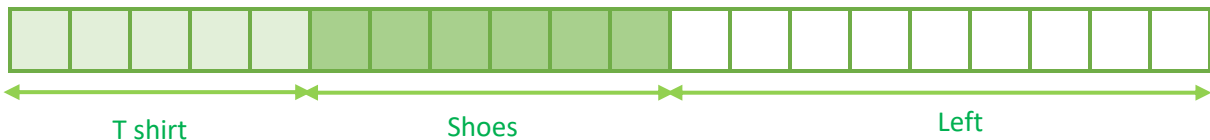
$$= 2$$

20-cent coins = $10u$		50-cent coins = $10u$
$= 10 \times 2$		$= 8 \times 2$
$= 20$		$= 16$

$$\begin{aligned} \text{John's money at end} &= 20 \times 0.2 + 16 \times 0.5 \\ &= 4 + 8 \\ &= \$12 \end{aligned}$$

John had \$12 at the end.

20. John had some money. He spent  $\frac{1}{4}$  of it on a T-shirt and  $\frac{2}{5}$  of the remainder on a pair of shoes. After that, his parents gave him \$120. The ratio of the total amount of money he had at the end to the amount of money he had at first was 5 : 4. How much did John have at first?



Money at first : Money at end

$$\begin{array}{ccc} 4 & : & 5 \\ 20 & : & 25 \end{array} \quad \leftarrow \times 5$$

Left =  $9u$  → before parents gave the \$120

$$\text{Parents Gave} = 120$$

$$\text{New total} - \text{left} = 120$$

$$25u - 9u = 120$$

$$16u = 120$$

$$1u = 120 \div 16$$

$$= 7.50$$

$$\begin{aligned} \text{John at first} &= 7.50 \times 20 \\ &= \$150 \end{aligned}$$

**John had \$150 at first.**